A Note on Scheduling Problems Arising in Satellite Communications *

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Abstract

We present a brief overview of recent results on scheduling in Time Division Multiple Access (TDMA) satellite communications systems which augment those included in a recent survey. These results address problems arising in Satellite Switched TDMA (SS/TDMA) systems and SS/TDMA systems with intersatellite links, as well as generalizations and relatives of these problems. They also have application in other areas, e.g. I/O in parallel computers.

Key words: scheduling, telecommunications, satellite communications, edge-coloring, heuristics, parallel computers, parallel I/O

Introduction

The recent paper by Prins[1] presents a good survey of scheduling problems arising in Time Division Multiple Access (TDMA) satellite communications systems. Readers interested in that paper may also be interested in several other results[2,3,4,5,6] not included in the survey[1] but, which, in some cases, are superior to those described there. The problems we have considered arise in Satellite Switched TDMA (SS/TDMA) systems and SS/TDMA systems with intersatellite links (ISL), as well as problems which are generalizations and relatives of those. We also mention some open questions regarding these problems.

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SS/TDMA systems

The basic SS/TDMA system described in Prins[1] consists of n earth stations communicating via a satellite which has $k \leq n$ repeaters and one on-board switch. The total traffic to be transmitted from station i to station j is known and specified as d_{ij} packets of fixed length, and m such transfers have to take place, i.e., there are m values of d_{ij} such that $d_{ij} > 0$. Each station can send or receive at most one packet during a time slot, but is permitted to send and receive at the same time. A switching mode consists of a configuration of the satellite switch which allows collision-free communication between up to k transmitting stations and k receiving stations. It takes $\tau \geq 0$ units of time to change between switching modes.(Note that Prins[1] uses the notation s for the number of stations, ν for the number of transfers, and m for the number of repeaters. We prefer the notation we have used as it corresponds more closely to that used by other researchers in the area and, as we will see, also to useful notions in graph theory.)

The fundamental problem is to find a schedule for transmitting the traffic from the senders to the receivers via the on-board switch. We are interested in the variant of the problem where switching time τ is negligible, so that the objective is to minimize the schedule length and a secondary objective is to minimize the number of switching modes. Following the terminology of Prins, this is problem *P1*. In particular, we are interested in the situation where preemption is allowed. We will consider both subcases of this problem, where $k = n$ and $k \leq n$. Borrowing from the terminology of Prins, we will call these problems *P1.A* and *P1.B*.

We also introduce a notation to define a problem *P0* which is a subset of *P1*. It is identical to *P1* except that no attention is paid to the secondary objective of minimizing the number of switching modes. Problem *P0* is relevant as switches become faster and it can be assumed that $\tau = 0$. Once again, problem *P0* has two subcases, *P0.A* (where $k = n$) and *P0.B* (where $k \leq n$).

This notation allows us to refer to the first algorithm described by Prins, which is an algorithm to solve *P0.A*. The algorithm is an open-shop scheduling algorithm by Gonzalez and Sahni[7] and runs in time $O(n^4)$. It is easy to show[3] that *P0.A* can be modeled as the problem of finding an optimal edge-coloring of a weighted bipartite graph, where now n is the number of vertices and $m = O(n^2)$ is the number of edges. (In fact, the Gonzalez-Sahni algorithm runs in time $O(m^2)$.)

P0.A can also be solved by algorithms which may be faster than that of Gonzalez and Sahni's in practice (these algorithms have not been mentioned in the survey[1].) Let K be the maximum number of packets to be sent to any station by any station, i.e., let $K = \max d_{ij}$. Then Gabow and Kariv[2] present an optimal algorithm to solve *P0.A* in time $O(nm \log K)$, i.e., $O(n^3 \log K)$. Jain et al[4] present a pseudo-polynomial[8] algorithm **A2** which solves *P0.A* in time $O(n^{0.5} mK \log n)$, which may be suitable when K is small. Recently Chen and Liu[9] have proposed an algorithm for *P0.A* where, essentially, batch admission control is applied at the inputs of the switch; during a batch, no input port is allowed to accept more than L packets for scheduling. With this restriction, they have developed an algorithm which runs in time $O(n^2 L)$.

The problem *P0.B*, where $k \leq n$, can be modeled as edge-coloring a weighted bipartite graph where at most k edges may receive the same color. The **KT** algorithm of Bongiovanni et al[10], mentioned in the survey, can be used to generate minimum-length schedules for *P0.B*, and runs in time $O(n^5)$. Jain et al[4] present algorithms which are also optimal but faster: algorithms **A1**, **A2** and **A3** which run in time $O(m^2 n^{0.5} + mn^{1.5})$, $O(Kmn^{0.5} \log n)$ and $O((n^2 + nm)(\log n + \log K))$ respectively. Thus while **A1** is the most generally suitable algorithm, **A2** is suitable when K is small and **A3** when K is large but bounded. Experimental comparisons of **KT** and **A1 - A3** have been presented[5].

The **KT** algorithm can be used for solving problem *P1.B*, where the secondary objective of minimizing the number of switching modes is also considered. Algorithms **A1 - A3** can also be used for solving *P1.B*, but it has not been determined how well they perform in meeting the secondary objective.

Another interesting question is whether simple heuristic algorithms can produce schedules which are close to optimal but which run in less time than the optimal algorithms. Heuristic algorithms are typically much easier to program and may be quite useful in practice, particularly for on-line scheduling or situations where real-time constraints are involved. For the problems *P0.A* and *P0.B*, Jain et al[11,6] show a range of simple greedy heuristic algorithms that produce schedules which are close to optimal (on average) in a fraction of the running time of the optimal algorithms. It remains an open question as to how many switching modes these heuristics produce.

Finally, the algorithms described so far have been centralized algorithms in the sense that the entire traffic matrix is known at some central processor where the scheduling algorithm is to be run. For some applications (e.g., scheduling I/O in parallel computers), this assumption may not hold, and distributed scheduling algorithms have been developed[12].

SS/TDMA systems with ISL

An interesting generalization of the SS/TDMA system is discussed by Prins[1], where there are two satellites connected via an ISL. Each station is covered by only one of the satellites, and each satellite now contains $k + 1$ repeaters, i.e., an extra repeater for communicating over the ISL. It is assumed that $k = n$, the number of stations covered by each satellite. Each ISL is a bidirectional full-duplex link allowing at most one packet to be sent in each direction at any given time, i.e., its capacity is one. The SS/TDMA with ISL problem is to schedule the traffic among the earth stations so as to minimize the schedule length.

Prins[1] describes a heuristic algorithm designed by Bertossi et al[13] which produces a schedule of at most twice the minimum schedule length, and runs in time $O(n^{4.5})$. Jain et al[3] describe a heuristic, based upon the **Tree** algorithm of Jain and Sasaki[14], which also produces a schedule of at most twice the minimum length, but runs in time $O(n^4)$. In addition, the heuristic can solve more general cases of the problem, where the earth-satellite links or the ISL have capacity greater than one, or where the terrestrial portion of the communication network has a hierarchical structure.

Generalizations and related problems

As pointed out by Prins[1], several generalizations of the problems discussed in the the survey have also been studied. Generalizations include situations in which preemption is not permitted, or preemption is permitted arbitrarily (i.e., not only at packet boundaries), where communication is via transceivers (devices which permit transmission and reception, but not both at the same time), and where the the communication network has a hierarchical switching structure (so as to improve utilization and extensibility of earth-satellite links).

Several of these problems, as well as existing algorithms to solve these and related open problems, have been previously described[3,5]. We have also described a more abstract and general model for specifying scheduling problems[3,5] which is useful for recognizing the underlying similarity of problems and algorithms which have been studied in different application areas such as satellite communications, parallel computer I/O operations and file transfers.

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